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Level Set Analysis of Two-Fluid Interfacial Flows

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Abstract

In this study, application of Level Set method in modeling incompressible immiscible two-phase flows for some benchmark problems such as rising bubble and bursting bubble at the free surface is explained. Derivations of Level Set function and convective terms are done using fifth order Weighted Essentially Non-Oscillatory (WENO) scheme. It is observed that Level Set method is very successful in modeling two-phase flows, especially in handling topological changes like bubble skirt pinching off and merging of bubble with the free surface.

Keywords: Level Set Method; Interface Modeling; Two-Phase Flows.

1. Introduction

Movement of clouds in the air, melting of the ice, fuel combustion in engine, boiling phenomenon in reactors, die casting of metals, solidification process in metallurgy and many other phenomena are just few examples of multiphase flows that happen in our daily lives and also in many important engineering and industrial applications. Therefore, analyzing of such these flows have a great importance. Although experimental analysis gives more realistic, accurate and reliable results, high costs of preparing experimental setups and time consuming experiments make less accurate but faster and more economic numerical methods preferable for many researchers in this field. In order to define material properties for each phase in the flow field, it is required to define an interface, which seperates different phase fields, and follow its evolution during the whole simulation. To this end, the Level Set function is applied.

2. Level Set Methodology

In order to model multiphase flows, it is required to define an interface and follow its evolution in entire domain. In Level Set method (LSM) [1, 2, 3], this interface is represented by zero level set of a continuous function such that,

$$\phi(\vec{x}, t) = \begin{cases} > 0 & \text{exterior} \\ 0 & \text{interface} \\ < 0 & \text{interior} \end{cases} \tag{1}$$

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To avoid having Level Set function be too steep or too flat, $\phi(\vec{x}, t)$, is set to be a distance function $\phi(\vec{x}, t) = d(\vec{x}, t) = min(|\vec{x} - \vec{x_I}|)$, where $\vec{x_I}$ shows the position of points on the interface and $d(\vec{x}, t)$ determines the distance of other points to the nearest points on the interface, i.e, $d(\vec{x}, t) = \sqrt{(x - x_I)^2 + (y - y_I)^2 + (z - z_I)^2}$ and $|\nabla\phi(\vec{x}, t)| = 1$. This Level Set function is called signed distance function.

Geometric properties of the interface such as normal vector, $\vec{n}$, and curvature, κ, can be easily calculated using Level Set function, ϕ, i.e, $\vec{n} = \frac{\nabla\phi}{|\nabla\phi|}$ and $\kappa = \nabla \cdot \vec{n}$. Propagation of $\phi(\vec{x}, t)$ is done by solving Level Set equation, $\frac{\partial\phi}{\partial t} + F_n|\nabla\phi| = 0$, in which F_n is a speed function normal to the interface and it varies depending on many factors such as the geometry of the problem, local curvature (κ), underlying velocity field that transports the interface, etc. For example, in the case of two-phase incompressible fluid flows, the speed function is the vlocity field solved from Navier-Stokes equations. Therefore, F_n is turned to be $\vec{V_n} = \vec{V} \cdot \vec{N}$ and since $\vec{N} = \frac{\nabla\phi}{|\nabla\phi|}$, Level Set equation is changed to

$$\frac{\partial\phi}{\partial t} + \vec{V} \cdot \nabla\phi = 0 \tag{2}$$

The obtained level Set function (ϕ) from equation(2) is not a signed distance function anymore. To avoid this situation, a so-called re-initialization step must be performed, in which the current level set function is replaced by a smoother and less distorted function which has the same zero level-set. The procedure is as the following:

$$d_\tau + sign(\phi)(|\nabla d| - 1) = 0 \tag{3}$$

With initial value $d(\vec{x}, 0) = \phi(\vec{x})$. Where $\phi(\vec{x})$ is the level Set function calculated from equation (2).

There are many variants of mollified signum function ($sign(\phi)$), one of them is due to Sussman et al.[3]

$$S(\phi) = \frac{\phi}{\sqrt{(\phi)^2 + \delta x^2}} \tag{4}$$

Another one is due to Peng et al.[4]

$$S(\phi) = \frac{\phi}{\sqrt{(\phi)^2 + |\nabla\phi|^2(\delta x)^2}} \tag{5}$$

In this study, $sign(\phi)$ is calculated due to equation suggested by Peng. The equation (3) is evolved untill meeting the steady state condition. Moreover, $\nabla\phi$ in equation (2) is calculated using Weighted Essentially Non Oscillatory (WENO) scheme [5]

3. Governing Equations

In this study, $2 - D$ fluid motion is considered. Behaviour of the fluids is governed by the incompressible Navier-Stokes equations defined in the domain $\Omega = \Omega_1 \cup \Omega_2 \cup \Gamma_f$, where Ω_1 denotes fluid phase 1 and Ω_2 denotes fluid phase 2 and $\Gamma_f = \partial\Omega_1 \cap \partial\Omega_2$ is the interface between two fluid phases:

$$\rho_i \frac{D\vec{u_i}}{Dt} = -\nabla p_i + \nabla \cdot (\mu_i S_i) + \rho_i \vec{g} \qquad \text{in } \Omega_i \tag{6}$$

$$\nabla \cdot \vec{u_i} = 0 \qquad \text{in } \Omega_i \tag{7}$$

$$\vec{u_i}|_\Gamma = 0 \qquad \text{in } \Omega_i \tag{8}$$

$$\vec{u_i}|_{t=0} = u_{0_i} \qquad \text{in } \Omega_i \tag{9}$$

Where $i \in 1, 2$ and material derivative, $\frac{D\vec{u}}{Dt}$, is equal to $\frac{D\vec{u}}{Dt} = \frac{\partial\vec{u}}{\partial t} + \vec{u}.\nabla\vec{u}$. The viscous stress tensor is given by: $S_i := \nabla\vec{u_i} + \{\nabla\vec{u_i}\}^T$. The surface tension boundary condition at the interface is given by:

$$(T_1 - T_2).\vec{n} = \sigma\kappa\vec{n} \tag{10}$$

Where $T_i = -p_i + \mu_i S_i$ denotes the stress tensor, σ is the surface tension coefficient, κ is the local curvature, and $\vec{n}$ denotes the surface normal on Γ_f. Furthermore, the velocity must be continuous across the free surface, i.e., $\vec{u_1} = \vec{u_2}$ on Γ_f. In order to solve equation (6) simoltaneously for both fluid 1 and fluid 2, it is required to redfine fluid properties, ρ and μ, as ϕ-dependant variables as the following, where H^ε denotes smoothed Heaviside function and $\delta^\varepsilon(\phi)$ is the associated smoothed delta functional. In this way, fluid properties vary smoothly across the interface of thickness ε, which is set to $1.5dx$ in this paper.

$$\rho^\varepsilon(\phi) = \rho_2 + (\rho_1 - \rho_2)H^\varepsilon(\phi) \qquad \text{and} \qquad \mu^\varepsilon(\phi) = \mu_2 + (\mu_1 - \mu_2)H^\varepsilon(\phi) \tag{11}$$

$$H^\varepsilon(\phi) = \begin{cases} 0 & \text{if} \quad \phi < -\varepsilon \\ \frac{1}{2}(1 + \frac{\phi}{\varepsilon} + \frac{1}{\pi}\sin(\frac{\pi\phi}{\varepsilon})) & \text{if} \quad |\phi| \leq \varepsilon \\ 1 & \text{if} \quad \phi > \varepsilon \end{cases} \tag{12}$$

$$\delta^\varepsilon(\phi) := \partial_\phi H^\varepsilon = \begin{cases} \frac{1}{2\varepsilon}\left(1 + \cos(\frac{\pi\phi}{\varepsilon})\right) & \text{for} \quad |\phi| < \varepsilon \\ 0 & \text{elsewhere} \end{cases} \tag{13}$$

Using continuum surface force (CSF) approach [6], surface tension boundary condition (10) is included in momentum equation (6) as a force term. By applying projection method [7], the following equations are obtained

$$\rho(\phi)\frac{D\vec{u}}{Dt} + \nabla p = \nabla \cdot (\mu(\phi)S) - \sigma\kappa(\phi)\delta(\phi)\nabla\phi + \rho(\phi)\vec{g} \tag{14}$$

$$\frac{\vec{u^\star} - \vec{u^n}}{\delta t} = -(\vec{u^n}.\nabla)\vec{u^n} + \vec{g} + \quad (\nabla \cdot (\mu(\phi^n)S^n) - \sigma\kappa(\phi^n)\delta(\phi^n)\nabla\phi^n) \tag{15}$$

$$\frac{\vec{u^{n+1}} - \vec{u^\star}}{\delta t} + \frac{\nabla p^{n+1}}{\rho(\phi^{n+1})} = 0 \tag{16}$$

$$\nabla \cdot \vec{u}^{n+1} = 0 \tag{17}$$

$$-\nabla \cdot \left(\frac{1}{\rho(\phi^{n+1})}\nabla\hat{p}^{n+1}\right) = -\nabla \cdot \vec{u}^\star \tag{18}$$

with $\hat{p}^{n+1} := \delta t p^{n+1}$.

Non-dimensional form of equation (14) is

$$\frac{\partial\vec{u}}{\partial t} = -\vec{u} \cdot (\nabla\vec{u}) - \frac{1}{\bar{\rho}(\phi)}\{\nabla p + \frac{1}{Re}(\nabla \cdot (\bar{\mu}(\phi)S)) - \frac{1}{Eo}(\kappa\delta\nabla\phi) + \vec{e}\} \tag{19}$$

Where Reynolds and Eotvos numbers are defined as $Re = \frac{\rho_1 VL}{\mu_1}$ and $Eo = \frac{\rho_1 V^2 L}{\sigma}$, L is a characteristic length, V is a characteristic velocity which is equal to $V = \sqrt{gL}$, and σ is surface tension coefficient. $\bar{\rho}$ and $\bar{\mu}$, non-dimensional density and viscosity, are defined as $\bar{\rho} = \frac{\rho}{\rho_1}$ and $\bar{\mu} = \frac{\mu}{\mu_1}$, where ρ_1 and μ_1 are base density and viscosity.

4. Results

Two different test cases, rising bubble and bursting bubble at the free surface are considered in order to show the ability of LSM in capturing moving interfaces. In the first test case, rising gas bubble in a surrounding liquid is considered. This is a well known and very important problem in multiphase flows for those dealing with bubbly flows. The Level Set function is defined as $\phi = \sqrt{(x - x_c)^2 + (y - y_c)^2} - R$. The lighter gas bubble experiences shape changes while it is rising due to buoyancy forces. The computational domain for Rising Bubble test case is set to $(0; 1) \times (0; 2)$ and the initial shape of bubble is supposed to be a circle with center $(0.5, 0.5)$ and radius $R = 0.25$. Free-slip and no-slip boundary conditions are imposed to vertical and horizontal walls respectively. Density and viscosity ratios are $\rho_1/\rho_2 = 1000$ and $\mu_1/\mu_2 = 100$ respectively. Reynolds number is set to $Re = 700$ and Eotvos number is set to, $Eo = 500$. The shape changes that bubble experiences during its elevation is depicted in figure (1-a) . Computations are done in 80×160 rectangular grid. Since Eotvos number is relatively high,

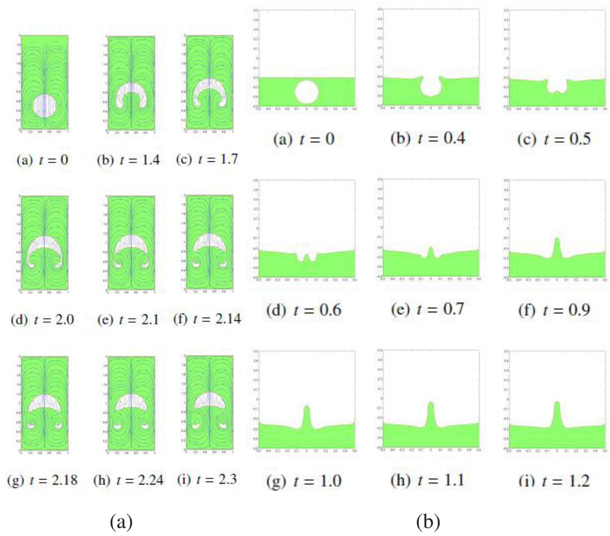

Fig. 1. a) Rising Bubble. b) Bursting bubble at the free surface

bubble will catch the skirted shape as time goes on. In high Reynolds numbers as in this test case, vortices that are created in the wake of bubble cause more intense circulations that leads to seperation of bubble skirts.

In the next test case, in order to show the ability of LSM in handling topological changes like merging of two interfaces, bursting bubble at the free surface problem is considered. In this case, bubble rises until it meets and merges with the free surface. Same as previous test case, Reynolds number, density and viscosity ratios are $Re = 700, \frac{\rho_1}{\rho_2} = 1000, \frac{\mu_1}{\mu_2} = 100$. Initially, free surface is located at $y_s = -0.2$, bubble has a radius of $R = 0.125$, its center is located at $(x_c, y_c) = (0, -0.35)$. The Level Set function is defined by defining two functions $\phi_1 = \sqrt{(x - x_c)^2 + (y - y_c)^2} - R$, $\phi_2 = y_s - y$ and taking the minimum of them $\phi = min(\phi_1, \phi_2)$.In this case, surface tension effects are neglected. It is observed that a fluid jet is formed after bubble merges with the free surface because of high pressure difference that occurs at a very small region as depicted in Figure (1-b).

5. Conclusion

Immiscible incompressible two-phase flows have been considered and results for some test cases, such as rising bubble and bursting bubble at the free surface, are presented. The interface, which seperates two different phases, is captured using LSM in which there is no need to reconstruct the interface after each time step and topological changes like merging or pinching off phenomena handled automatically. However, it has been observed that the mass (or area) conservation is not that much satisfactory, which is the only drawback of this method.

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